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Case study

A new model for the optimization of periodic inspection intervals with failure interaction: A case study for a turbine rotor

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ABSTRACT

Inspection is one of the important activities to detect and fix failures in repairable system. Optimization of inspection intervals has critical role in maintenance cost and operating system. When a component fails, it is renewed or repaired. A great deal of periodic inspection research is for hidden failure and considered one of the perfect (renew) and minimal repair policies. In literature, the lack of simultaneous consideration of both perfect and minimal repair in reliability model has been observed. This study presents new reliability model by synchronous consideration of both minimal and perfect repair. As well as, the expected total maintenance cost is presented and modeled to figure out the optimal inspection interval. The proposed model is more comprehensive model in reliability evaluation and can be applied in different pertinent problems. The model is applied to steam turbine system which the rotor considered as soft component and filter as hard component. The result revealed that the system should be inspected every 12 month.

1. Introduction

A steam turbine is a system that extracts thermal energy from pressurized steam and uses it to carry out mechanical operation on a rotating output shaft. In center of turbine, the moving rotor generates power. When rotor fails (erosion), the turbine works but with lower performances (losses profit). The failure of rotor just detected in inspection. The steam rotor inspection has great cost. The time for rotor inspection and repair may extend to more than one month.

The cost of maintenance is one of the major performance indexes in manufacturing and operation. The aim of maintenance is to maximizing reliability and minimizing cost [1–3]. The optimization costs can be considered as returning lost profit or potential budget injection in maintenance [1]. Also, repairing and inspections of system increases maintenance costs. In contrast, increasing inspection and maintainability reduces the downtime penalty cost. The inspection is one of the important activities to detect and fix failures in repairable system. Optimization of inspection intervals has critical role in decreasing total costs of maintenance system, reduces inspection costs and increases the performance of operating system.

For soft components or systems, in which failures are only detected at the time of inspection (hidden failures), it is important to determine the optimal inspection time. Inspection policies for systems with hidden failures (soft system) can be divided into periodic and non-periodic inspections. If the time between successive inspections has been randomly

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determined with different inspection times, it is known as non-periodic inspection [4]. For periodic inspections, the interval is constant and is called Failure Finding Interval [5]. Various researchers have reached significant results for a periodic inspection policy. Periodic inspection is easier to schedule in practice than a sequential inspection (non-periodic inspection).

A great deal of periodic inspection research for hidden failure addresses the cost issue. In these works, the optimum inspection interval and maintenance policy are obtained by minimizing the expected cost in a given time period [6]. A number of periodic inspection models have been proposed regard this cost criteria. Generally, they employed one or several of the following assumptions: (1) inspections are performed at interval T, and inspections do not degrade the system; (2) The failure time of the system has a cumulative distribution $F(t)$, and its failure is detected only at the inspections; (3) The time required for inspection is negligible; (4) Repair is conducted immediately if failure is detected, and repair makes the system "renew" (perfect repair).

Barlow et al. [4] introduced a basic model, for determining the optimal inspection interval to minimize the expected cost. Accordingly, in their model, only two costs are included: an inspection fixed cost, and a loss (downtime) cost per unit time resulting from the elapsed time between failure detection and the next inspection. Based on Barlow's model, some researchers have investigated algorithm to determine an optimum inspection time, such as Luss [7], Schultz [8] and Alfares [9].

While the above models only considered inspection cost and loss cost, other research takes the repair/replacement cost into account. Nakagawa [10] considers an age-based inspection policy for a single standby unit with some extension of the basic model. His model assumes that both inspection and repair will renew the system.

Tahgipour and Banjevic [11] investigated the optimal inspection interval for a multi-unit repairable system to minimize expected cost over a finite and infinite time horizon. Ahmadi and Kumar [12] developed a cost rate function model to determine the optimum inspection interval time and frequency of inspection and restoration of an aircraft's repairable components.

Transportation systems, such as airplanes and trains, consist of different components in a mixed configuration, which are called multi-component systems. In a multi-component system, the components may interact with one another (for example load sharing). These interactions create dependency among the components that can be categorized as follows [13]:

1. **Economic dependence**, occurs when the cost of maintenance and replacement produce dependency among the components. In other words, the replacement of a number of components together may cost less than the replacement of them individually. In fact, it might be cost efficient to replace a functional component at the time of replacing some failed components, which creates dependency among them. Systems like aircrafts, ships, telecommunication, and mass production lines are examples of this type [14]. This type of dependency is focus of the present study.
2. **Structural dependence**, occurs when the maintenance and replacement of some components require replacement or disassembly of some other parts or components [13].
3. **Stochastic/probabilistic dependence**, which happens when the state of a component, such as its workload, affects the life-time distribution of the other components. Stochastic dependence is sometimes referred to as probabilistic dependence or failure interaction. For instance, the failure of one component increases the failure/hazard rate of other components [13].

Although there are many studies [15–17] on reliability and maintenance of multi-component systems, most of these studies consider only one of the dependencies discussed above. The complex nature of the problem makes these models too complicated to be solved or analyzed when considering more than one of these dependencies [18]. In 1986, Thomas [13] put together a survey reviewing the models which were previously proposed for complex systems alongside their maintenance and replacement policies. In 1991, Cho and Parlar [19] reviewed the maintenance of various multi-component models. In 2011, Sarkar et al. [20] reviewed the literature and collected different maintenance policies for complex systems. This research provides a good overview for both single and multi-component systems during the past 50 years. According to these reviews [13,19,20], there are several publications on multi-component systems with economic dependence, though little attention has been devoted to systems with stochastic dependence. Most of these studies only consider two-component systems, because in practice it is difficult and sometimes impossible to evaluate the actual effect of the failure of multiple components on each other [21]. Murthy and Nguyen [15,16] studied the maintenance of systems considering stochastic dependence. They formulated the failure interactions between components in a two and multi-component systems and developed expressions for the expected operation costs for both finite and infinite life-times. Scarf and Deara [22] developed a model considering both economic and stochastic dependences between components in a two-component system. The policies in their model were age-dependent. They extended their model to block replacement policies for a two component system [23]. Zequeira and Bérenguer [17] analyzed the maintenance costs for a two-component standby parallel system taking into account the stochastic dependence. Periodic inspections and preventive maintenance were at the center of their study. Taghipour et al. [24] proposed a model to find the optimal periodic inspection interval on a finite time horizon for a complex repairable system. They considered costs consist of inspection, repair, and downtime penalty cost. Inspection interval with failure interaction in two and multi component system have been studied by GolMakani and Moakedi [25,26]. In their two-component studies, they took account capacitor bank (first component) and the transformer (second component) for a distribution substation in an electric power distribution system. Taghipour and Kassaei [27] studied the both Stochastic and economic dependency. They studied failure interaction for k-out-of-n load-sharing system. They assumed each time a component fails, distributed load to the remaining components, as well as an extra load which increases the hazard rates of the remaining components. Recently, Rezaei and Imani [1] proposed new risk based inspection optimization model by considering fuzzy failure interaction. They assumed, the system can be worse along failure occur to failure detection (next inspection) and followed the minimal repair policy. They also, studied optimization inspection interval under perfect repair policy [28].

In literature, the lack of simultaneous consideration of both replacement (renew) and repair (minimal repair) in reliability model made us conduct this study. In this paper, the perfect and minimal repair policy is considered in reliability and cost models. As well as, the cost model includes inspection, repair, and loses cost (downtime penalty cost). The present work considers stochastic dependence for system. The novelty of this paper is to present new reliability model by considering both minimal repair and replace simultaneously in reliability and cost models. As well as, the expected total maintenance cost is presented to figure out the optimal inspection interval. This proposed model is more comprehensive model in reliability and cost evaluation and can be used in different problems where the assumptions made in the model are applicable.

In Section 2, the inspection optimization model, problem definition, and the proposed model are presented. In Section 3, the numerical example is solved.

2. Inspection optimization model

2.1. Problem definition

Here, a repairable system consists of rotor and filter with failure interaction is considered. The failure of the rotor is soft and the filter is hard. The hard failure causes the system stop but the soft failures does not. The rotor is periodically inspected and if an erosion failure is observed during the inspection, it may minimally or perfectly repaired (due to failure situation). The soft failure detects just in inspection. The perfect repair restores system as good as new and the minimal repair restores system as bad as old. The replacement and repair without error considered as perfect repair. Thus, for the rotor, there is a time delay between a real occurrence of failure and its detection. The failure of rotor decreases the performance in each time unit. So, long time delay have greater profit losses (or greater cost) to the turbine. The hard failure of the filter is detected immediately as soon as it occurs and perfectly repaired. Also, it is assumed that the second component is not inspected. The perfect repair of filter restores it to as good as new ones but for rotor does not. It is assumed that the first component's (rotor) failures have an increasing failure rate (non-homogeneous Poisson process; NHPP) and the second component's failures have a constant failure rate (homogeneous Poisson process; HPP). The parameter of p usually is uncertain. In order to overcome this uncertainty, the expert judgment is used [29]. In addition to the above assumptions, the following assumptions are hypothesized:

1. The inspections of the first component are perfect, i.e. they diagnose the soft failure of the first component without any error.
2. The inspection time of the second component are ignored and treated as being zero.
3. The soft failure of the first component cannot convert to hard failure.
4. The cost resulting from the second component includes only the cost associated with its perfect repair. Since the failure rate of this component is assumed constant, its corresponding cost per unit time is constant and, thus, it is not included in the optimization model.

In this proposed model, the rotor (soft failure part) is indicated by number one and that the filter (hard failure part) is indicated by number two. The defined parameters and variables are encapsulated in Table 1.

2.2. Proposed model

As noted, the second component (filter) failures accelerates the failure of the first component (rotor), but the first component failure does not affect the second component failure. The parameter of p indicates accelerating effect of second component to first component and is not synergistic. The component 1 and 2 refers to soft (rotor) and hard (filter) components, respectively. The $\lambda_1^j(x)$ is given by:

$$\lambda_1^j(x) = \lambda_1(x|N_2(x) = j) = (1 + jp)\lambda_1^0(x), j = 0, 1, \ldots \tag{1}$$

where, $\lambda_1^0(x)$, is the failure rate of the first component, if the second component does not fail until time x. The distribution function of first component assumed as Weibull distribution.It is assumed that the second component's failures occur according to HPP with a constant failure rate. Thus, we have

$$p(N_2(x) = j) = \frac{(\lambda_2 \times x)^j e^{-\lambda_2 \times x}}{j!}, j = 0, 1, \ldots \tag{2}$$

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Table 1
Parameters and variables definitions.

$\lambda_1(x)$	The average failure rate of the first component at time x	C_1^s	The cost of each inspection of the first component
$\lambda_1^j(x)$	The failure rate of the first component at time x, provided that the number of failures of the second component from the beginning of the planning horizon until time x is equal to j; $j = 0, 1, 2, \ldots$	C_1^{MR}	The minimal repair cost
		C_1^{PR}	The perfect repair cost
$((k-1)\tau, k\tau]$	kth inspection interval in the cycle T, $k = 1, 2, \ldots, n$	C_1^D	The downtime penalty cost associated with the first component per each unit of elapsed time from the soft failure of the first component to its detection at the Inspection time
$N_2(x)$	A random variable representing the number of failures of the second component from the beginning of the planning horizon until time x	λ_2	The failure rate of the second component
$E[C_1^T]$	The expected total cost of the first component in the cycle T	p	The present of failure interaction from hard component to soft component.
$E\left[C_1^{(k-1)\tau,k\tau}\right]$	The expected total cost of the first component in kth inspection interval of the cycle T, i.e. From a scheduled inspection at $k\tau$ over time period $((k-1)\tau, k\tau]$.	T	The planning horizon length (e.g. one year) which is known and fixed
τ	The time between two consecutive inspections, $\tau = T/n$	n	The number of inspections to be performed on the first component during the cycle T
$p_{\tau,1}^k(t)$	The probability that the first component dose does not fail in kth inspection interval of the cycle T with τ inspection interval (reliability function), provided that we know that its age at the beginning of the cycle T is equal to t and that it is not as good as new at that time	t	The initial age of the first component at the beginning of the cycle T

The expected failure rate of the first component at time x, $\lambda_1(x)$, depends on the number of the second component's failures $j = 0,1,2,\ldots$. Thus, from Eqs. (1) and (2), $\lambda_1(x)$, is given by

$$\lambda_1(x) = \sum_{j=0}^{\infty}\lambda_1(x|N_2(x)=j)\times p(N_2(x)=j) = \sum_{j=0}^{\infty}[\lambda_1^0(x)(1+jp)]\times\frac{(\lambda_2\times x)^j e^{-\lambda_2\times x}}{j!}$$
$$= e^{-\lambda_2\times x}\lambda_1^0(x)\left[\sum_{j=0}^{\infty}\frac{(\lambda_2\times x)^j}{j!}+p\sum_{j=0}^{\infty}\frac{j(\lambda_2\times x)^j}{j!}\right] = e^{-\lambda_2\times x}\lambda_1^0(x)[e^{\lambda_2\times x}+p(\lambda_2\times x)e^{\lambda_2\times x}] = \lambda_1^0(x)[1+p(\lambda_2\times x)] \quad (3)$$

The cumulative distribution function is given by Eq. (4), and it is simplified by following equations, Eq. (5)–(7).

$$F_1(x) = 1-e^{-\int_t^{t+x}\lambda_1(x)dx} \quad 0\le x\le\tau \quad (4)$$

$$F_1(x) = 1-e^{-\int_t^{t+x}\frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1}[1+p(\lambda_2\times x)]dx} = 1-e^{-\frac{1}{\theta^\beta}\int_t^{t+x}\beta x^{\beta-1}+p\lambda_2 x^\beta dx} \overset{t=0}{=} 1-e^{-\frac{1}{\theta^\beta}\left(x^\beta+p\lambda_2\frac{x^{\beta+1}}{\beta+1}\right)} \quad (5)$$

Assume that

$$\zeta = \frac{1}{\theta^\beta}, \quad \zeta' = \frac{p\lambda_2}{\beta+1} \quad (6)$$

Then Eq. (6) given by

$$F_1(x) = 1-e^{-\zeta\left(x^\beta+\zeta' x^{\beta+1}\right)} \quad (7)$$

The optimal inspection interval time determines for cycle T (planning horizon) which is fixed. Inspections take place at times $k\tau(\tau, 2\tau, \ldots, k\tau)$, as well, repair are performed at the end of the cycle T (for $k = n$, at the time $n\tau$. The objective is to find the optimal inspection interval time to minimize the expected total cost of the system over the cycle T. When the component fails, it remains in a failed state until the next inspection time. Therefore, if the component failed in each inspection interval, a downtime penalty cost is incurred. The cost is proportional to the elapsed time from failure time to its detection at inspection time. Thus, the costs for resulting from the system in each of the inspections k, $k = 1,2,\ldots,n$ includes the cost of inspection, C_1^s, the cost of repair if found failed, C_1^{PR} (or C_1^{MR}), and the penalty cost for the elapsed time for the failure, C_1^D, Thus, the expected cost incurred in the inspection k for each cycle (τ) is given by

$$\underset{\forall\,\tau=T,T/2,\ldots,1}{E_\tau[C_1^T]} = ((\text{expected cost of inspection}) + (\text{expected cost of minimal repair/perfect replace})$$

$$+ (\text{expected cost of downtime}))$$

$$= \sum_{k=1}^{T/\tau} E_\tau[C_1^{(k-1)\tau,k\tau}] = \left(\left(\sum_{k=1}^{T/\tau} C_1^s \right) + \sum_{k=1}^{T/\tau} ((e^{b_1\tau})C_1^{MR} + (1-e^{b_1\tau})C_1^{PR})[1-p_{\tau,1}^k(t)] + \sum_{k=1}^{T/\tau} C_i^D[\tau(1-p_{\tau,1}^k(t))] \right) \tag{8}$$

The $p_{\tau,1}^k(t)$ is probability that the component 1 does not fail in kth inspection interval of the cycle T with τ inspection interval, provided that we know that its age at the beginning of the cycle T is equal to t. The τ is expressed in months and 1 month is always regarded as minimum value of τ. The $p_{\tau,1}^k(t)$ as well called reliability function. Due to different inspection intervals times, the probability of $p_{\tau,1}^k(t)$ is depends on $p_{\tau,1}^{k-1}(t)$. To obtain $p_{\tau,1}^k(t)$, the Bayesian theory is used by Eq. (9).

$$p_{\tau,1}^k(t) = p_{\tau,1}^k(t|\text{safety in } p_{\tau,1}^{k-1}(t))p_{\tau,1}^{k-1}(t) + p_{\tau,1}^k(t|\text{unsafety in } p_{\tau,1}^{k-1}(t))(1-p_{\tau,1}^{k-1}(t)), \quad k = 1,\ldots,T/\tau \tag{9}$$

Soft component can be minimal or perfect repairs after failure. The perfect repair like replacement restores component as new but minimal repair does not. The probability that the component 1 does not fail in kth inspection interval if failed in last interval covers by Bayesian theory as follows;

$$p_{\tau,1}^k(t|\text{unsafety in } p_{\tau,1}^{k-1}(t)) = p_{\tau,1}^k(\left[t|\text{unsafety in } p_{\tau,1}^{k-1}(t)\right]|\text{minimal repair in last inspection})\times p(\text{minimal repair in last inspection})$$
$$+ p_{\tau,1}^k(\left[t|\text{unsafety in } p_{\tau,1}^{k-1}(t)\right]|\text{perfect repair in last inspection})\times p(\text{perfect repair in last inspection}) \tag{10}$$

In proposed model, the perfect repair policy selects with probability of $\int_0^\tau r(\mathrm{x})$ and minimal policy with $1-\int_0^\tau r(\mathrm{x})$. The failure time and inspection presented in Fig. 1.

From Eqs. (9) and (10) the structure of reliability model presented as Eq. (11):

$$p_{\tau,1}^k(t) = p_{\tau,1}^{k-1}(t)\left[1-F_1(x)|_{(k-1)\tau}^{k\tau}\right]$$
$$+ (1-p_{\tau,1}^{k-1}(t))\left[\int_0^\tau r_1(x) \int_{(k-2)\tau}^{(k-1)\tau} \frac{\beta}{\theta}\left(\frac{y}{\theta}\right)^{\beta-1} e^{-\left(\frac{y}{\theta}\right)^\beta}\left[1-F_1(x)|_y^{k\tau}\right]dy + \left(1-\int_0^\tau r_1(x)\right)\left[1-F_1(x)|_{(k-1)\tau}^{k\tau}\right]\right] \tag{11}$$

In Eq. (11), assumed that system can be worse from failure time occurrence to failure detection (next inspection). So, the reliability from failure occurrence time to its detection is considered. For the different inspection intervals, the $F(x)|_{(k-1)\tau}^{k\tau}$ indicates the probability of the soft component failure at $((k-1)\tau, k\tau]$ interval, when in the last interval the soft component has safety condition. As mentioned above, the soft failure detects and repairs in inspections. The $r(x)$ (exponential distribution) is

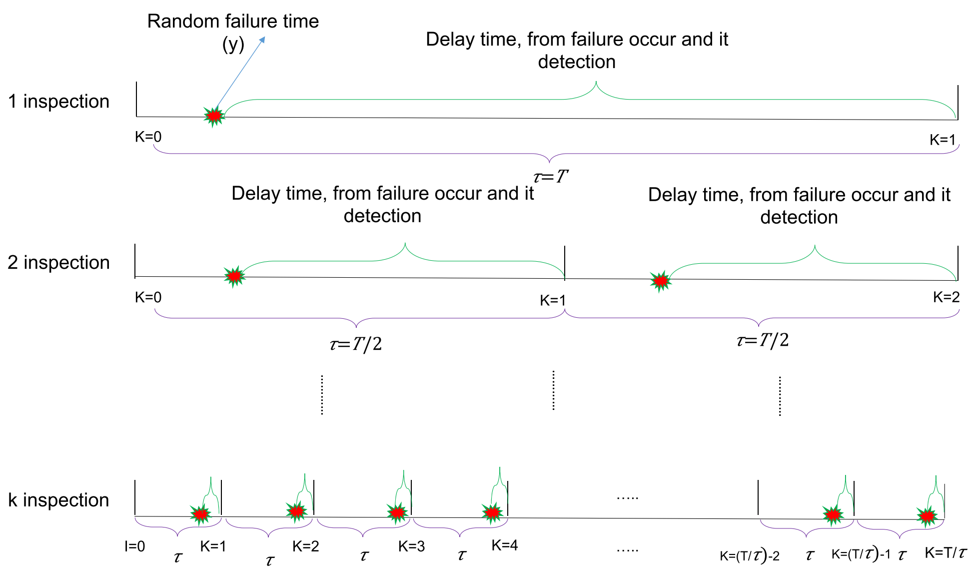

Fig. 1. Structure of inspection and failure times.

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perfect repair distribution function of component 1. When $\int r(x) = 1$ and $0 \le \int r(x) < 1$ perfect and minimal repair should be done respectively. According to Eq. (7):

$$p_{\tau,1}^{k}(t) = p_{\tau,1}^{k-1}(t)\left[e^{-[\zeta((k\tau)^{\beta}+\zeta'(k\tau)^{\beta+1})-\zeta(((k-1)\tau)^{\beta}+\zeta'((k-1)\tau)^{\beta+1})]}\right] + (1-p_{\tau,1}^{k-1}(t))\left[(1-e^{-b_1\tau})\int_{(k-2)\tau}^{(k-1)\tau}\frac{\beta}{\theta}\left(\frac{y}{\theta}\right)^{\beta-1}e^{-\left(\frac{y}{\theta}\right)^{\beta}}\left[e^{-[\zeta((k\tau)^{\beta}+\zeta'(k\tau)^{\beta+1})-\zeta((y)^{\beta}+\zeta'(y)^{\beta+1})]}\right]dy + e^{-b_1\tau}e^{-[\zeta((k\tau)^{\beta}+\zeta'(k\tau)^{\beta+1})]}\right] \tag{12}$$

In Eq. (12), the b is the perfect repair rate. An example for 2 inspection frequency is given by:

$$p_{T/2,1}^{1}(t) = 1\times\left[e^{-\left[[\zeta((T/2)^{\beta}+\zeta'(T/2)^{\beta+1})-\zeta(((1-1)T/2)^{\beta}+\zeta'((1-1)T/2)^{\beta+1})]\right]}\right] + (1-1)\times[\ldots] = e^{-[\zeta((T/2)^{\beta}+\zeta'(T/2)^{\beta+1})]} \tag{13}$$

$$p_{T/2,1}^{2}(t) = p_{T/2,1}^{1}(t)\left[e^{-\left[\zeta((T)^{\beta}+\zeta'(T)^{\beta+1})-\zeta((T/2)^{\beta}+\zeta'(T/2)^{\beta+1})\right]}\right] + (1-p_{T/2,1}^{1}(t))\left[\left(1-e^{-b_1T/2}\right)\int_{0}^{T/2}\frac{\beta}{\theta}\left(\frac{y}{\theta}\right)^{\beta-1}e^{-\left(\frac{y}{\theta}\right)^{\beta}}\left[e^{-\left[\zeta((T)^{\beta}+\zeta'(T)^{\beta+1})-\zeta((y)^{\beta}+\zeta'(y)^{\beta+1})\right]}\right]dy + e^{-b_1T/2}e^{-\left[\zeta((T)^{\beta}+\zeta'(T)^{\beta+1})\right]}\right] \tag{14}$$

3. Case study

In this paper, a steam turbine rotor from Iran is studied as a two-component system. The failure of rotor is scrutinized based on erosion or corrosion. If the types of material ingested into the machine are chemically reactive, especially involving the metal in the turbine parts, the result is corrosion. There are two classifications of corrosion in gas turbines: cold corrosion and hot corrosion. Cold corrosion occurs in the compressor due to wet deposits of salts, acids, steam, aggressive gases such as chlorine, sulfides, or perhaps oxides. This may result in reducing cross sectional properties by removal of material over an area or concentrated corrosion culminated in pitting (Fig. 2). The results of corrosion can be very similar to erosion, with the exception that corrosion can also intrude into cracks and metallurgical abnormalities to accelerate other damage initiation mechanisms. These corrosion effects are irreversible just like erosion. The only way to bring the blades back to the original condition is with replacement.

Hot corrosion occurs in the turbine area of the gas turbine. This section is exposed to materials that may intrude not just from the air, but also from the fuel or water/steam injection which can be difficult to filter. These include metals such as sodium, potassium, vanadium, and lead that react with sulfur and/or oxygen during combustion. After combustion, these will deposit themselves on combustor liners, nozzles, turbine blades, and transition pieces and cause the normally protective oxide film on these parts to oxidize several times faster than without it. The examples of rotors and their blades erosion failure are presented in Fig. 3.

In turbine system, the repair time and replacement are non-negligible, and repairing time may take a month or longer. Sometimes the replacement may be more cost-effective due to downtime penalty cost.

Effective filtration can require several filter stages to remove different materials from the air, or to remove more particles, different phases (solid, liquid), or smaller particles (Fig. 4). Filters to remove rain and snow, mist, smoke or dust, and finer particles all require variations in filter design. These particles has effect on blades failure if filter failed. The failure of filter

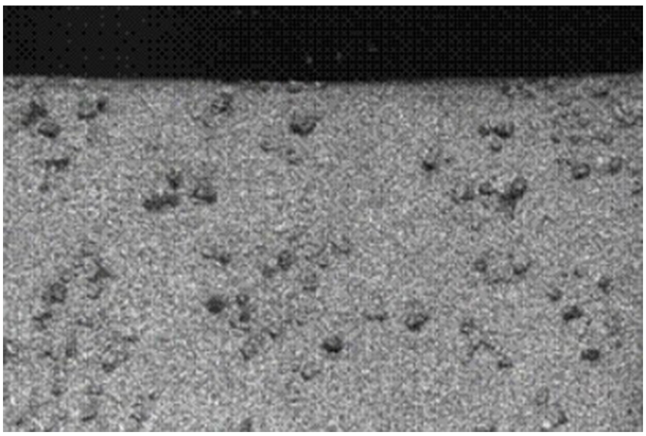

Fig. 2. Corrosion/pitting on blade.

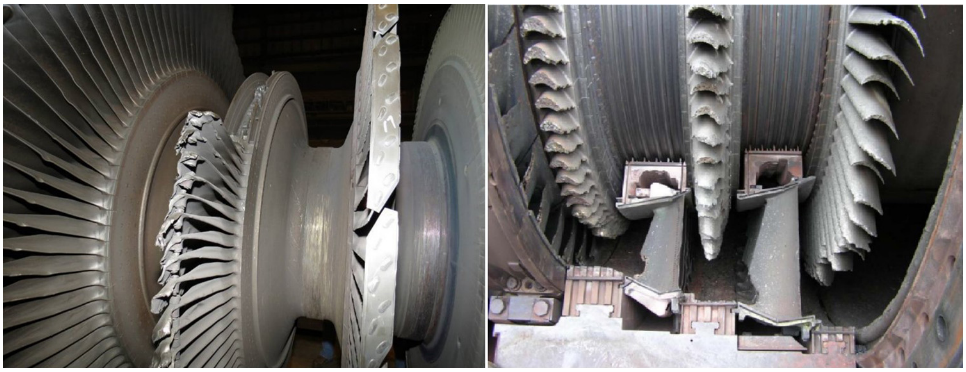

Fig. 3. Example of erosion failures.

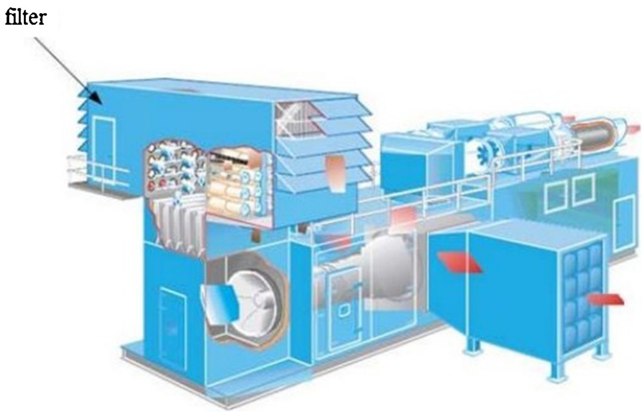

Fig. 4. The structure of turbine.

increases the failure rate of rotor (blades failure). In this study, rotor considered as soft component and filter as hard component. When the turbine rotor fails, the performance of turbine decreases. Its failures detected just in inspections.

In this case, the hazard rate of the rotor as $\lambda_1^0(x) = \beta/\theta(x/\theta)^{\beta-1}$, where $\beta = 4.9$, $\theta = 20$, and other parameters are as follows: $T = 36$, $\lambda_2 = 0.7$, $b = 0.9$, $\xi = 0.85$, $p = 0.6$, $b = 0.5$, $C^s = 50000$, $C^{MR} = 500000$, $C^{PR} = 1500000$ and the downtime penalty cost is

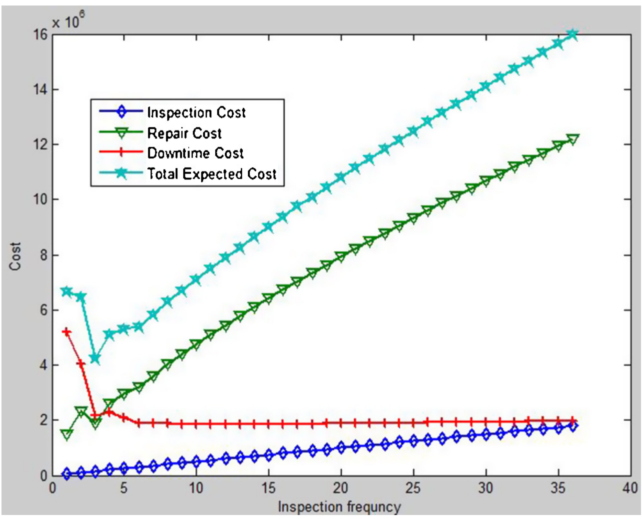

Fig. 5. The costs resulting for different inspection frequencies $\tau = 36, 18, \ldots, 1$.

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Table 2
The $p^k_{\tau,1}(t)$ results for $\tau = 36, 18, \ldots, 1$ and different k.

Inspection intervals	Sub-inspection intervals				
	$K=1$	$K=2$	$K=3$	$K=4$	$K=5$
$\tau=36$	2.7e−28				
$\tau=18$	0.2563	0.1906			
$\tau=12$	0.8592	0.122	0.7525		
$\tau=9$	0.9677	0.2872	0.6822	0.3042	
$\tau=36/5$	0.9899	0.6767	0.3364	0.639	0.3476

From Eq. (8), total expected cost for each fixed inspection interval time (τ) is shown in Fig. 5. The optimal interval is regard to $\tau = 12$.

$C^p = 144000$ per unit time (monthly). To determine Weibull parameters, the maximum-likelihood estimation (MLE) is used [30]. The costs currency is Tomans of Iran.

The expected total cost is calculated by the proposed model in which MATLAB software (version 2015) is employed to increase the correctness of calculation. The Simpson rule [1] is used for reliability calculations. To indicate reliability analysis, the summary results of $p^k_{\tau,1}(t)$ for component 1 is presented. The $p^k_{\tau,1}(t)$ values for component 1 is proposed in Table 2.

Increasing the number of inspections increases the inspection costs and reduces the downtime penalty cost. The contrast between these two costs caused the Non-strict total cost plot. In riskless model, the optimal inspection interval is obtained for 3 inspection frequencies and it's related to $\tau = 12$.

4. Conclusion

The failure detection in soft component is totally more difficult and important in comparison with hard component. In system include one soft component, the failure detection time can be records from performance decreased time. But, in system with multi-soft-component, detecting failure time is very difficult. The reliability analysis applied for detection failure time. In literature, generally, researchers considered one of the minimal and perfect repair policies in reliability and cost modeling.

To find optimal inspection interval in soft system, the downtime and inspection costs are another index in addition to reliability.

In this study, a comprehensive model to reliability evaluation is presented in accordance with minimal and perfect repair policies. According to the achieved result, for constant k, the reliability increases by decreasing the interval time. Besides, for constant τ, the reliability reduced by increasing the inspection time.

This model is applied to steam turbine rotor as a soft component and filter as a hard component. The result indicates the system should be inspected every 12 month.

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